# CHARGE RADII OF THE PROTON AND VALENCE QUARKS IN THE QUASIPOTENTIAL MODEL

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**Abstract**

The mean square radius of the proton charge distribution was studied in the framework of the relativistic quasipotential quark model in assumption of the SU(6)-symmetry. It was shown that the proton charge radius is represented as function $f(m/\gamma)/M^2$ for point quarks ($m$ is a quark mass, $\gamma$ is the scale of the coupling energy, $M$ is the nucleon mass, $f$ is undimentional function, which does not depended on $M$). It was shown, that in the ultra relativistic region the mean square radius of the bound system can have negative value. To describe simultaneously nucleon magnetic moments and proton radius in the oscillator model it is need to suppose that quark has the negative mean square radius $\langle r_{1q}^2\rangle = -1.875/m^2$.

## Introduction.

In works [1, 2, 3] the formalizm of the quark model in the quasipotential approach was considered: the nucleon quark wave function was constructed in the impulse approximation [1], the calculations of the magnetic moments [2] and the ratio of the axial and vector coupling constants [3] were performed. It was obtained that model has property which is followed by the relativistic kinematics of quarks: nucleon magnetic moments (in units of nuclear magnetons) and ratio $G_A/G_V$ depend on one undimensional parameter, which is constructed as ratio of the quark mass and the energy scale parameter, and don't depend on the ratio of the quark mass and nucleon mass. In this case the valent qaurk mass and energy of coupling are intrinsic parameter, which are related. Nucleon charge radius, considered in the this paper, has this property.

## 1. Basic relations of the model

In the quasipotential quark model [1] the electromagnetic current in the rest frame of the initial nucleon has the form [2]:

$$\langle \mathbf{K}\lambda'_J\lambda'_T \mid J_\mu(0) \mid \mathbf{0}\lambda_J\lambda_T\rangle \equiv eJ_\mu^{\lambda'_J\lambda_J}(\mathbf{K},\mathbf{0})\delta_{\lambda'_T\lambda_T} =$$
$$= 3e\int d\Omega_{\mathbf{p}_1}d\Omega_{\mathbf{p}_2}\varphi(\overset{o}{M}{}'_0)G_\mu\varphi(M_0)\delta_{\tau'\tau}, \tag{1}$$

where

$$\overset{o}{M}{}'_0 \equiv E_{\overset{o}{\mathbf{p}}_1} + E_{\overset{o}{\mathbf{p}}_2} + E_{\overset{o}{\mathbf{p}}{}'_3}, \qquad M_0 \equiv E_{\mathbf{p}_1} + E_{\mathbf{p}_2} + E_{\mathbf{p}_3},$$
$$G_\mu^{(3)} \equiv \frac{1}{E_{\overset{o}{\mathbf{p}}{}'_3}E_{\mathbf{p}_3}}(\chi^{\lambda'_J\lambda'_T}B^1B^2j_\mu^{(3)}B^3\chi^{\lambda_J\lambda_T}), \tag{2}$$
$$B^{(1)}_{\{\lambda'_{s_k}\lambda'_{t_k}\lambda_{s_k}\lambda_{t_k}\}} \equiv \left[\overset{+}{D}{}^{(1/2)}{}_{\lambda'_{s_1}\lambda_{s_1}}(L_{\overset{o}{\mathbf{p}}_{12}}, \overset{\not o}{\mathbf{p}}_1\ )\,\overset{+}{D}{}^{(1/2)}{}_{\lambda'_{s_2}\lambda_{s_2}}(L_{\overset{o}{\mathbf{p}}_{12}}, \overset{\not o}{\mathbf{p}}_2\ )\right]\delta_{\lambda'_{t_1}\lambda_{t_1}}\delta_{\lambda'_{t_2}\lambda_{t_2}}\delta_{\lambda'_{t_3}\lambda_{t_3}}\delta_{\lambda'_{s_3}\lambda_{s_3}},$$
$$B^{(2)}_{\{\lambda'_{s_k}\lambda'_{t_k}\lambda_{s_k}\lambda_{t_k}\}} \equiv \left[D^{(1/2)}_{\lambda'_{s_1}\lambda_{s_1}}(L^{-1}_{\mathbf{K}},\mathbf{p}_1)D^{(1/2)}_{\lambda'_{s_2}\lambda_{s_2}}(L^{-1}_{\mathbf{K}},\mathbf{p}_2)D^{(1/2)}_{\lambda'_{s_3}\lambda_{s_3}}(L^{-1}_{\mathbf{K}},\mathbf{p}'_3)\right]\delta_{\lambda'_{t_1}\lambda_{t_1}}\delta_{\lambda'_{t_2}\lambda_{t_2}}\delta_{\lambda'_{t_3}\lambda_{t_3}},$$
$$B^{(3)}_{\{\lambda'_{s_k}\lambda'_{t_k}\lambda_{s_k}\lambda_{t_k}\}} \equiv \left[D^{(1/2)}_{\lambda'_{s_1}\lambda_{s_1}}(L_{\mathbf{p}_{12}}, \overset{\not o\!\!/}{\mathbf{p}}_1\ )D^{(1/2)}_{\lambda'_{s_2}\lambda_{s_2}}(L_{\mathbf{p}_{12}}, \overset{\not o\!\!/}{\mathbf{p}}_2\ )\right]\delta_{\lambda'_{t_1}\lambda_{t_1}}\delta_{\lambda'_{t_2}\lambda_{t_2}}\delta_{\lambda'_{t_3}\lambda_{t_3}}\delta_{\lambda'_{s_3}\lambda_{s_3}},$$
$$ej_\mu^{(3)\,\lambda'_{s_3}\lambda_{s_3}}(\mathbf{p}'_3,\mathbf{p}_3)\delta_{\lambda'_{t_3}\lambda_{t_3}} \equiv \langle\mathbf{p}'_3\lambda'_{s_3}\lambda'_{t_3}|\hat{j}_\mu^{(3)}(0)|\mathbf{p}_3\lambda_{s_3}\lambda_{t_3}\rangle.$$

Here $j^{(3)}_{\mu}(0)$ is a current of third quark. For convenience the electron charge $e$ is written as factor. Sum of 3-momenta in the wave function of the relative motion $\varphi$ is equal to zero: $\overset{o}{\mathbf{p}}_1 + \overset{o}{\mathbf{p}}_2 + \overset{o\prime}{\mathbf{p}}_3 = 0$, $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$. The list of notations is given at the end of the section. The current normalization condition $J_0(\mathbf{0},\mathbf{0}) = e_N$ ($e_N$ is the nucleon charge in units of the electron charge $e$) leads to the WF normalization condition [1], which is given by:

$$\int d\Omega_{\mathbf{p}_1} d\Omega_{\mathbf{p}_2} \mid \varphi_M(M_0) \mid^2 / E_{\mathbf{p}_3} = 1. \tag{3}$$

The Dirac form factors of the nucleon are defined by the expression:

$$J^{\lambda'_J\lambda_J}_{\mu}(\mathbf{K},\mathbf{0}) = e_N \bar{U}^{\lambda'_J}(\mathbf{K})\{\gamma_\mu F_1(Q^2) + \frac{i\kappa}{2M}F_2(Q^2)\sigma_{\mu\nu}q_\nu\}U^{\lambda_J}(\mathbf{0}), \tag{4}$$

where $\kappa$ is the nucleon anomalous magnetic moment, $\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$, $Q^2 \equiv -q^2 > 0$, $q = (P' - P)$, $P' = (E^M_{\mathbf{K}}, \mathbf{K})$, $P = (M, \mathbf{0})$. Sachs form factors is convenient for analysis of the experimental data: $G_E(Q^2) = F_1(Q^2) + F_2(Q^2)\kappa Q^2/4M^2$, $G_M(Q^2) = F_1(Q^2) + \kappa F_2(Q^2)$. Let write the zero component of the third quark current like (4), but in form of Pauly $\sigma-$matrix, with zero quark anomalous magnetic moments:

$$j^{(3)}_0(\mathbf{p}'_3, \mathbf{p}_3) = \frac{\hat{e}_q f_1(Q^2_{(3)})}{2\sqrt{(E_3+m)(E'_3+m)}}[(E_{\mathbf{p}_3}+m)(E_{\mathbf{p}'_3}+m) + (\sigma\mathbf{p}'_3)(\sigma\mathbf{p}_3)], \tag{5}$$

where $\hat{e}_3$ is charge operator of the third quark in units of the electron charge, $f_1$ is the quark Dirac form factor, depended on $Q^2_{(3)} = -(p'_3 - p_3)^2$. Quark radius is defined by decomposition over $Q^2_{(3)}$: $f_k(Q^2_{(3)}) \approx 1 - \langle r^2_{kq}\rangle Q^2_{(3)}/6$.

In work [1] the relativistic three-particle oscillator was proposed in the effective mass approximation. This approximation is defined by the condition: $m^{eff}_k - \langle E_{\mathbf{p}_k}\rangle << m^{eff}_k$, where $m^{eff}_k = \sqrt{m^2 + \langle p^2_k\rangle}$, $\langle p^2_k\rangle = \int d\Omega_{\mathbf{p}_1} d\Omega_{\mathbf{p}_2} |\varphi_M|^2 p^2_k / E_{\mathbf{p}_3}$, $\langle E_{\mathbf{p}_k}\rangle = \int d\Omega_{\mathbf{p}_1} d\Omega_{\mathbf{p}_2} |\varphi_M|^2 E_{\mathbf{p}_k}/E_{\mathbf{p}_3}$. The wave function $\varphi$ of relative motion in this approximation is written as:

$$\varphi^{osc}_M(\{\overset{o}{\mathbf{p}}_k\}) = N\exp[-\frac{m}{\gamma^2}(M_0 - 3m)] \approx N\exp(-\frac{\mathbf{k}^2}{\gamma^2} - \frac{3\mathbf{k}'^2}{4\gamma^2}) \approx \tag{6}$$

$$\approx N\exp(-\frac{1}{\gamma^2}(\overset{o}{\mathbf{p}}{}^2_1 + \overset{o}{\mathbf{p}}{}^2_2 + \overset{o}{\mathbf{p}}_1\overset{o}{\mathbf{p}}_2)\frac{2m}{m + m^{eff}}).$$

where $\mathbf{k} = (\pi_1 - \pi_2)/2$, $\mathbf{k}' = (\pi_1 + \pi_2)/3 - 2\pi_3/3$; $\pi_k = \overset{o}{\mathbf{p}}_k \sqrt{2m/(m + E_{\overset{o}{\mathbf{p}}_k})}$ is half-momentum [4]; the energy has the form $E_{\overset{o}{\mathbf{p}}_k} = m + \pi^2_k/2m$. In the effective mass limit $\pi_k \approx \overset{o}{\mathbf{p}}_k \sqrt{2m/(m + m^{eff})}$, $\pi_1 + \pi_2 + \pi_3 \approx 0$, $E_{\overset{o}{\mathbf{p}}_k} \approx m + \overset{o}{\mathbf{p}}{}^2_k/(m + m^{eff})$. If $\langle p^2_k\rangle$ is small, then $m^{eff} \approx m$ and effective mass approximation passes to nonrelativistic limit.

Here we give a list of notations: $d\Omega_{\mathbf{p}_k} = d\mathbf{p}_k/E_{\mathbf{p}_k}$, $E_{\mathbf{p}_k} = \sqrt{\mathbf{p}^2_k + m^2}$, $E^M_K = \sqrt{\mathbf{K}^2 + M^2}$, $\mathbf{K}$ and $\mathbf{p}_k$ are nucleon and quark momenta, $M$ and $m$ are nucleon and quark masses, $U$ and $u$ are nucleon and quark spinors, $\{\mathbf{p}_k\lambda_{s_k}\lambda_{t_k}\} \equiv \mathbf{p}_1\lambda_{s_1}\lambda_{t_1}\mathbf{p}_2\lambda_{s_2}\lambda_{t_2}\mathbf{p}_3\lambda_{s_3}\lambda_{t_3}$; $\chi$ is SU(3) nucleon wave function. The total spins and isospins of the nucleon and constituent quarks are omitted and equal to 1/2; $\lambda_J$, $\lambda_{s_k}$ are the third projections of the nucleon and quark spins and equal to +1/2. We assume a sum over the repeated indices of the third projection of spin and isospin. The normalization condition of vector states and spinors of nucleon and quarks are given by:

$$\langle K'J'\lambda'_J T'\lambda'_T \mid KJ\lambda_J T\lambda_T\rangle = (2\pi)^3\delta^3(\mathbf{K}' - \mathbf{K})\delta_{\lambda'_J\lambda_J}\delta_{\lambda'_T\lambda_T},$$

$$\langle \mathbf{p}'_k s_k \lambda'_{s_k} t_k \lambda'_{t_k} \mid \mathbf{p}_k s_k \lambda_{s_k} t_k \lambda_{t_k} \rangle = (2\pi)^3 E_{\mathbf{p}_k} \delta^3(\mathbf{p}'_k - \mathbf{p}_k) \delta_{\lambda'_{s_k}\lambda_{s_k}} \delta_{\lambda'_{t_k}\lambda_{t_k}},$$

$$\bar{u}^{\lambda_{s_k}}(\mathbf{p}) u^{\lambda_{s_k}}(\mathbf{p}) = m, \;\; \bar{U}^{\lambda_J}(\mathbf{K}) U^{\lambda_J}(\mathbf{K}) = M/E^M_{\mathbf{K}} \text{ (there is no sum over } \lambda_{s_k} \text{ and } \lambda_J).$$

$\overset{\not o}{\mathbf{p}}_k = L^{-1}_{\mathbf{p}_{12}} p_k$, where $p_{12} = p_1 + p_2$; $\overset{o}{p}_k = (L^{-1}_K p_k)$, $\overset{\not o}{p}_k = (L^{-1}_{\overset{\not o}{p}_{12}} \overset{o}{p}_k)$, where $\overset{o}{p}_{12} = \overset{o}{p}_1 + \overset{o}{p}_2$; $L^{-1}_K$ is the Lorentz boost to the rest frame of the nucleon and $L^{-1}_{\overset{\not o}{p}_{12}}$ and $L^{-1}_{p_{12}}$ are the Lorentz boosts to the rest frame of [1+2] quarks. Wigner rotation matrix has the form (see [1] and references there in):

$$D^{(1/2)}(L^{-1}_K, p_k) = \frac{(E_{\mathbf{p}_k} + m)(E^M_K + M) - (\sigma\mathbf{K})(\sigma\mathbf{p}_k)}{\sqrt{2(E_{\mathbf{p}_k} + m)(E^M_K + M)(E_{\mathbf{K}} E_{\mathbf{p}_k} - \mathbf{K}\mathbf{p}_k + mM)}}. \tag{7}$$

4-momentum $\overset{o}{p}_k$ in the c.m.s. has the components:

$$E_{\overset{o}{\mathbf{p}}_k} = \frac{1}{M}(E^M_{\mathbf{K}} E_{\mathbf{p}_k} - \mathbf{K}\mathbf{p}_k), \quad \overset{o}{\mathbf{p}}_k = \mathbf{p}_k - \frac{\mathbf{K}}{M}(E_{\mathbf{p}_k} - \frac{\mathbf{K}\mathbf{p}_k}{M + E^M_{\mathbf{K}}}). \tag{8}$$

## 3. Analitycal analysis.

By using the current (5), the expression for $G_E(Q^2)$ is represented as: $G_E(Q^2) = \sqrt{2E^M_{\mathbf{K}}/(M + E^M_{\mathbf{K}})} J_0(\mathbf{K}, \mathbf{0})$. Electric nucleon radius, defined according to the decomposition $G_E(Q^2) \approx e_N - \langle r^2_E \rangle Q^2/6$, expresses as:

$$\langle r^2_E \rangle = -\nabla^2_{\mathbf{K}} G_E(Q^2)|_{\mathbf{K}=0} = -\frac{3e_N}{4M^2} - \nabla^2_{\mathbf{K}} J_0(\mathbf{K}, \mathbf{0})|_{|\mathbf{K}=0}. \tag{9}$$

In the nonrelativistic limit we have from (9) and (1), denoting $e_N = 3(\chi, e^{(3)}_q \chi)$:

$$\langle r^2_E \rangle^{NR} = e_N \int d\mathbf{p}_1 d\mathbf{p}_2 (-i\nabla_{\mathbf{K}})^2 \varphi(\overset{o}{\mathbf{p}}_1, \overset{o}{\mathbf{p}}_2, \overset{o}{\mathbf{p}}{}'_3) \varphi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3). \tag{10}$$

Using the last expression of (6) with $m^{eff} \approx m$, we have from (10):

$$\langle r^2_E \rangle^{NR} = e_N \int d\mathbf{p}_1 d\mathbf{p}_2 |\varphi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)|^2 \left[ -\frac{\mathbf{p}^2_3}{\gamma^4} + \frac{2}{\gamma^2} \right]. \tag{11}$$

Now if we return to variables $\mathbf{k}$ and $\mathbf{k}'$, assuming $\mathbf{p}_1 = \mathbf{k} + \mathbf{k}'/2$, $\mathbf{p}_2 = -\mathbf{k} + \mathbf{k}'/2$ and using expression for the relative motion wave function (6) in variables $\mathbf{k}$ and $\mathbf{k}'$, we have well-known results [5]: $\langle r^2_E \rangle^{NR} = e_N/\gamma^2$, where ortonormalization condition (3) is used also. Now we can rewrite general expression for radius in the relativistic case with the oscillator wave function (6):

$$\langle r^2_E \rangle = -\frac{3e_N}{4M^2} + \int d\Omega_{\mathbf{p}_1} d\Omega_{\mathbf{p}_2} |\varphi|^2 \left[ -e_N \frac{m^2}{\gamma^4} (\nabla_{\mathbf{K}} \overset{o}{M}{}'_0)^2 \frac{1}{E_{\mathbf{p}_3}} + e_N \frac{m}{\gamma^2} \nabla_{\mathbf{K}}{}^2 \overset{o}{M}{}'_0 \frac{1}{E_{\mathbf{p}_3}} + \right.$$

$$\left. + 3\frac{m}{\gamma^2} (\nabla_{\mathbf{K}} \overset{o}{M}{}'_0)(\nabla_{\mathbf{K}} G_0) - 3\nabla_{\mathbf{K}}{}^2 G_0 \right]_{|\mathbf{K}=0} = e_N [\frac{-3}{4M^2} + \langle r^2_I \rangle + \langle r^2_{II} \rangle + \langle r^2_{III} \rangle + \langle r^2_{IV} \rangle]. \tag{12}$$

Four terms $\langle r^2_{I,II,III,IV} \rangle$ correspond to four expressions under integral in the expression (12). After differentiation over $\mathbf{K}$ for $\mathbf{K} = 0$ we don't have any $\sigma$-matrix, that is why all expressions are proportional to the nucleon charge $e_N = 3(\chi, e^{(3)}_q \chi)$. So, in the quark model the neutron radius equal to zero, but neutron form factors deviate from zero

for unzero transfer momenta [6]. Further we consider only the proton radius. Expressions for $\langle r_I^2\rangle$ and $\langle r_{II}^2\rangle$ have the forms:

$$\langle r_I^2\rangle = -\frac{1}{M^2}\frac{m^4}{\gamma^4}\int d\Omega_{\mathbf{p}_1} d\Omega_{\mathbf{p}_2} |\varphi|^2 \frac{1}{E_{p_3}} \frac{[\sum_{i=1}^3 E_{p_i}]^2}{E_{p_3}^2}\frac{\mathbf{p}_3^2}{m^2} \tag{13}$$

$$\langle r_{II}^2\rangle = \frac{1}{M^2}\frac{m^2}{\gamma^2}\int d\Omega_{\mathbf{p}_1} d\Omega_{\mathbf{p}_2} |\varphi|^2 \frac{1}{E_{p_3}}\left[3(E_{p_1}+E_{p_2}) + \frac{\mathbf{p}_3^2}{E_{p_3}^2}(E_{p_3}-E_{p_1}-E_{p_2})\right]\frac{\sum E_{p_i}}{E_{p_3} m} \tag{14}$$

Third term $\langle r_{III}^2\rangle$ contains derivative from $G_0$ (2). In this expression $B^{(3)}$ does not depend on $\mathbf{K}$. It was shown numerically that expression $\nabla_{\mathbf{K}} B^{(1)}|_{\mathbf{K}=0}$ is negledibly small in the wide regionof the parameters and $\nabla_{\mathbf{K}} B^{(1)}|_{\mathbf{K}=0}$ is proportional to $\overset{o'}{\mathbf{p}}_1 + \overset{o'}{\mathbf{p}}_2 = 0$ in the effective mass limit. Taking into account that $\mathbf{p}_1+\mathbf{p}_2 = -\mathbf{p}_3$, it is possible to show that contribution of $B^{(2)}$ is equal to zero. So, we have only contribution from $1/E_{\overset{o'}{\mathbf{p}}_3}$ and from the quark current:

$$\langle r_{III}^2\rangle \approx \frac{1}{M^2}\frac{m^2}{\gamma^2}\int d\Omega_{\mathbf{p}_1} d\Omega_{\mathbf{p}_2} |\varphi|^2 \frac{1}{E_{p_3}}\Big[E_{p_3}-E_{p_1}-E_{p_2}\Big]\frac{\sum E_{p_i}}{2mE_{p_3}}\frac{\mathbf{p}_3^2}{E_{p_3}^2} \tag{15}$$

Nonrelativistic expression for radius $(e_N/\gamma^2)$ can be obtained from (13)-(15), assuming $M \approx 3m$.

Relativistic term $\langle r_{IV}^2\rangle$ is represented as series of the negative powers $[m/\gamma]^2$ (including the zero power). As a result we have:

$$\langle r_E^2\rangle \quad = \quad \frac{9}{M^2}\Big(\frac{m^4}{\gamma^4}a_{-4} + \frac{m^2}{\gamma^2}a_{-2} + a_0 + \frac{\gamma^2}{m^2}a_2 + \frac{\gamma^4}{m^4}a_4 + ...\Big). \tag{16}$$

Therefore we don't have dimensional terms in the form $1/m^2$ for radius (16) (compare [5, 7]).

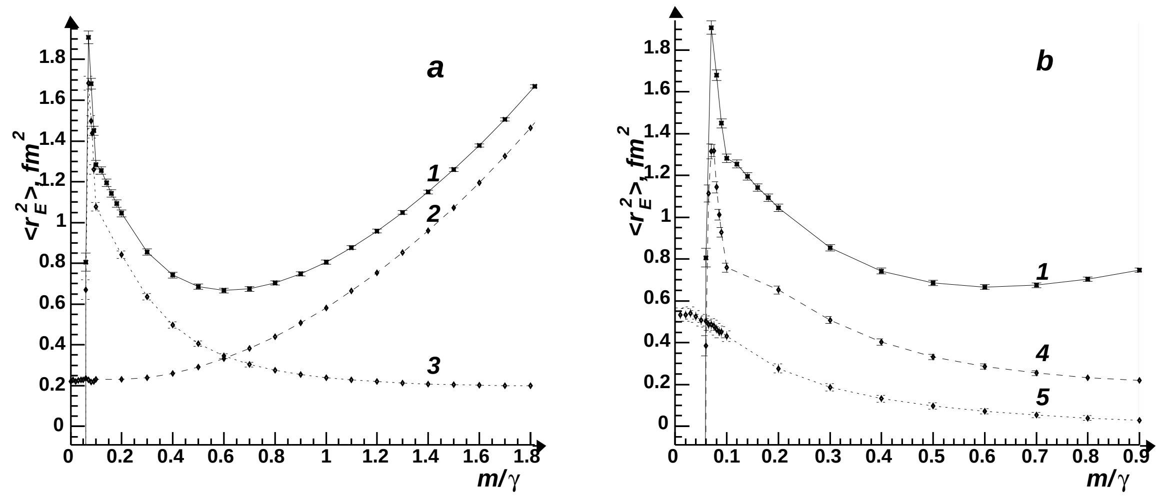
Dependence on the nucleon momentum $\mathbf{K}$ in (1) is defined by Lorentz-transformation, which contains 3-velosity $\mathbf{K}/M$ (see (8) ). Therefore, the nucleon radius (9) for arbitrary wave function is defined as $f(m/\gamma)/M^2$, where $f$ is undimensional function.If quarks have structure, then $f$ depends on the parameters of the quark structure.

## 3. Numerical analysis.

Figure $a$ shows dependences of proton radius $\langle r_E^2\rangle$ (curve 1), $\langle r_I^2\rangle + \langle r_{II}^2\rangle + \langle r_{III}^2\rangle$ (curve 2) and $\langle r_{IV}^2\rangle$ (curve 3) from $m/\gamma$. Points for $\langle r_E^2\rangle$ was obtained as follows: data for current and for electric form factor $G_E(Q^2)$ were obtained for fixed $m/\gamma$ and in the interval $0.001 < Q^2 < 0.1$ GeV$^2$. Integral was calculated by VEGAS [8] with five iteration and $10^6$ trials in each. After that points of $G_E(Q^2)$ were fitted by function $G_E(Q^2) = b_0 - b_1 Q^2/6 + b_2 Q^4 + b_3 Q^6 + b_4 Q^8$ for obtaining the radius $\langle r_E^2\rangle \equiv b_1$. Errors of the fit are shown on the figures.

For calculation of $\langle r_I^2\rangle + \langle r_{II}^2\rangle + \langle r_{III}^2\rangle$ we can use (13)-(15), calculated by VEGAS [8]. Error of the calculation is small and shown on the figure. For calculation of $\langle r_{IV}^2\rangle$ it was difficultly to obtain analitycal formulae, like (13)-(15). However, we can separate the contribution of $\langle r_{IV}^2\rangle$ in the current (1). For this it is sufficiently to replace $\varphi(\overset{o'}{M}_0) \to \varphi(M_0)$ and perform calculation as in case of $\langle r_E^2\rangle$.

If $m/\gamma \to \infty$ then $\langle r_E^2\rangle$ increases because of there are positive powers of $m/\gamma$, as in nonrelativistic theory. If $m/\gamma$ decreases then relativistic term with negative powers of $m/\gamma$ dominates.




From figure $a$ we can see that $\langle r_I^2\rangle + \langle r_{II}^2\rangle + \langle r_{III}^2\rangle$ has nonzero limit for $m/\gamma \to 0$. Indeed, it is possible to represent integrals in (13), (14), (15) as infinite series of the decomposition with the negative powers of $m/\gamma$. This decompositions have nonzero terms $[m/\gamma]^{-4}$ in (13) and $[m/\gamma]^{-2}$ in (14) and (15), which define limits $m/\gamma \to 0$ for $\langle r_I^2\rangle$, $\langle r_{II}^2\rangle$ $\langle r_{III}^2\rangle$. Nonzero limit of strong coupling for the proton radius is relativistic effect. In the nonrelativistic case size of system decreases to zero when the bound energy increases.

From figure $a$ we can see, that behaviour of the radius of the bound system is more complicated in the limit of strong coupling as compare to the behaviour of $\langle r_I^2\rangle + \langle r_{II}^2\rangle + \langle r_{III}^2\rangle$. In the region of small $m/\gamma$ form factor $G_E(Q^2) > 1$ for small $Q^2$, radius of the bound system is negative and its module increases when $m/\gamma \to 0$. We can analyze what terms in the expression (1) correspond to this behaviour of the form factor in the ultrarelativistic region. From figure $a$ we can see that the cause is the term $\langle r_{IV}^2\rangle$. According to (1), (2) contribution of $\langle r_{IV}^2\rangle$ consist of following parts: contribution of the Wigner rotation matrix $\langle r_D^2\rangle$ and contribution of the quark current $\langle r_j^2\rangle$. Cross term in which first derivative $\nabla_{\mathbf{K}}$ acts to Wigner rotation matrix, second derivative acts to the quark current, does not give any contribution (see explanation before (15)). Figure $b$ shows the proton radius dependences on $m/\gamma$: $\langle r_E^2\rangle$ (curve 1), $\langle r_j^2\rangle$ (curve 4), $\langle r_D^2\rangle$ (curve 5). It is possible to conclude, that the positive peak and the negative proton radius for small $m/\gamma$ is originated from the quark current. So, the reason for the negative radius of the bound system of the spinor particle is the dependence of the valence quark current on quark momenta, defined by expression (5).

The quark radius contributes to the terms with negative powers of $m/\gamma$ in the expression (16). We can write down the quark radius contribution to the proton radius exactly.

$$\Delta\langle r_E^2\rangle = \frac{9}{M^2}\langle r_{1q}^2\rangle m^2 \int d\Omega_{\mathbf{p}_1} d\Omega_{\mathbf{p}_2} |\varphi|^2 \frac{1}{E_{p_3}} \frac{m^2 + 2E_{p_3}^2}{3E_{p_3}^2} \Big(\frac{\sum_{i=1}^3 E_{p_i}}{3m}\Big)^2 \tag{17}$$

In work [2] for oscillator model (6) it was shown that simultaneous description of proton and neutron magnetic moments with 2% accuracy is obtained for $m/\gamma = 1.818$. For this value proton radius is more than experimental data $\sqrt{\langle r_E^2\rangle} = 0.870 \pm 0.008$ fm [9] and is equal to $\sqrt{\langle r_E^2\rangle} = 1.291 \pm 0.003$ fm. For simultaneous description of experimantal data it is need to include negative contribution into the proton mean square radius, which is

equal to $\Delta\langle r_E^2\rangle = -0.910$ fm$^2$ for experimental data $\sqrt{\langle r_E^2\rangle} = 0.870 \pm 0.008$ fm. Assuming that this contribution is related to the valence quark radius, we have $\langle r_{1q}^2\rangle m^2 = -1.875$.

Notice, that nonzero quark radius does not influence on the nucleon magnetic moments, by using of which the ratio $m/\gamma = 1.818$ was obtained. If quark does not have size, then value, obtained in this model is corresponding to experimental data for $m/\gamma = 0.4$ and $m/\gamma = 0.9$ (see figure a). For this $m/\gamma$ it is possible to describe the nucleon magnetic moments but with less accuracy ($m/\gamma = 0.4$: deviations are 11% for proton and 20% for neutron), ($m/\gamma = 0.9$: deviations are 4.5% for proton and 11% for neutron); or in other case if we introduce large the quark anomalous magnetic moments, which should be different for $u$- and $d$-quarks [2]. So, if we assume that good description of the proton and neutron magnetic moments with the zero quark anomalous magnetic moments is not accidental, then model indicates that the valence quarks must have the negative radius.

**Conclusion.**

So, in this model the nucleon radius is defined by undimensional parameters $m/\gamma$ and $\langle r_1^2 q\rangle m^2$. Absolute values of the quark characteristics (the quark mass and the quark radius) are not defined without specification of the "scale"-parameter, which defined the coupling energy. Oscillator wave function was used here for calculation, but the radius expression (16) with factor $1/M^2$ is general; it is possible that additional powers of $m/\gamma$ can change series (16) for other form of wave function and other definition of parameter $\gamma$. Dimensional factor $1/M^2$ is consequence of relativistic kinematic of the relative quark motion (see (8) and (9)). Scale invariant dependence of the nucleon radius and magnetic moments on $m/\gamma$ is related to that the wave function and quasipotential are independent on bound system mass $M$ in the impulse aproximation [1]. In the general case the relativistic expression for radius of the bound system contains infinite series of negative powers of $m/\gamma$. In the oscillator model this gives large negative radius of the system with strong coupling of the spinor particles. Wigner rotation matrices gives small contributions into the proton radius in the region of parameters, which correspond to the the experimental data of the proton radius. In this model it is need to intoduce negative quark radius for simultaneous description of the proton radius and nucleon magnetic moments.

# References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